ALGORITHM 14
COMPLEX EXPONENTIAL INTEGRAL

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procedure   EKZ(x,y,k,ε,u,v,n)  ;  real x,y,k,ε,u,v  ;
              integer n  ;
comment     EKZ computes w(z,k) = u + iv = z^k e^z ∫_z^∞ e^-t dt/t^k
            from the continued fraction representation found
            in H. S. Wall, Continued Fractions, Chap. 18 (D.
            Van Nostrand, New York, 1948). Input parameters
            are x, y, k, and ε where z=x+iy. Successive con-
            vergents are computed as follows: For n = 2, 3, 4,
            ..., D_n = z/(z + M × D_{n-1}), R_n =
            (D_n - 1)R_{n-1}, C_n = C_{n-1} + R_n, where M is
            k + (n-2)/2 or (n-1)/2 according to whether n
            is even or odd, and D_1 = R_1 = C_1 = 1. Computa-
            tion is stopped when C_n and C_{n-1} agree to the sig-
            nificance specified by ε. The corresponding index
            n is available after use of the procedure. This
            method is valid in the entire complex plane except
            for the origin and the negative real axis. Conver-
            gence is too slow to be practical for | z | < .05.
            Also for some range within the half-strip | y | < 2,
            x < 0 (this range depends on k). The method is
            valid for complex k, but only real k is considered
            in this procedure;
begin       real t1, t2, t3, M, K, c, a, d, b, g, h, ε1  ;
              integer m  ;
            comment R = a + ib,  D = c + id,  C = g + ih  ;
              ε1 := ε↑2  ;
            u := c := a := 1  ;  v := d := b := 0  ;
              n := 1  ;  K := k - 1  ;
BACK:       g := u  ;  h := v  ;  n := n + 1  ;
              m := n ÷ 2  ,
            if 2 × m = n then M := m + K else M := m  ;
            t1 := x + M × c  ;  t2 := y + M × d  ;
              t3 := t1↑2 + t2↑2  ;
            c := (x × t1 + y × t2)/t3  ;
              d := (y × t1 - x × t2)/t3  ;
            t1 := c - 1  ;  t2 := a  ;
              a := a × t1 - d × b  ;  b := d × t2 + t1 × b  ;
            u := g + a  ;  v := h + b  ;
            if (a↑2 + b↑2)/(u↑2 + v↑2) > ε1 then go to
              BACK  ;
end         EKZ
```

CERTIFICATION OF ALGORITHM 14
COMPLEX EXPONENTIAL INTEGRAL (A. Beam, *Comm. ACM*, July, 1960)

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EKZ was programmed by hand for the Royal-Precision LGP-30 computer, using a 28-bit mantissa floating-point interpretive system (24.2 modified). To facilitate comparison with existing tables (National Bureau of Standards Applied Mathematics Series 51 and 37), the real and imaginary parts of $E_k(z)$ were computed from u and v. Results are shown in the following table. In all cases, the values agreed with tabulated values within the tolerance specified.

x	y	k	ε	n
1×10^{-8}	1.0	1	10^{-1}	7
1×10^{-8}	1.0	1	10^{-2}	14
1×10^{-8}	1.0	1	10^{-3}	24
1×10^{-8}	1.0	1	10^{-4}	37
1×10^{-8}	1.0	1	10^{-5}	52
1×10^{-8}	1.0	1	10^{-6}	70
1×10^{-8}	1.0	1	10^{-7}	90
1×10^{-8}	1.0	1	10^{-8}	114
1×10^{-8}	2.0	1	10^{-6}	37
1×10^{-8}	3.0	1	10^{-6}	26
1×10^{-8}	4.0	1	10^{-6}	21
1.0	1×10^{-8}	1	10^{-6}	40
1.0	1.0	1	10^{-6}	34
1.0	2.0	1	10^{-6}	26
1.0	3.0	1	10^{-6}	21
2.0	1×10^{-8}	1	10^{-6}	23
2.0	1.0	1	10^{-6}	22
2.0	2.0	1	10^{-6}	20
2.0	3.0	1	10^{-6}	17
3.0	1×10^{-8}	1	10^{-6}	17
3.0	1.0	1	10^{-6}	17
3.0	2.0	1	10^{-6}	16
3.0	3.0	1	10^{-6}	15
4.0	0.0	0	10^{-6}	20
4.0	0.0	1	10^{-6}	15
4.0	0.0	2	10^{-6}	16
4.0	0.0	3(1)14	10^{-6}	17
4.0	0.0	15, 16	10^{-6}	16

It thus appears that the algorithm gives satisfactory accuracy, but that in certain ranges of the variables, the time required may be excessive for extensive use.