

## ALGORITHM 63

## PARTITION

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```

procedure partition (A,M,N,I,J); value M,N;
      array A; integer M,N,I,J;

```

**comment** I and J are output variables, and A is the array (with subscript bounds M:N) which is operated upon by this procedure. Partition takes the value X of a random element of the array A, and rearranges the values of the elements of the array in such a way that there exist integers I and J with the following properties:

```

M ≤ J < I ≤ N provided M < N
A[R] ≤ X for M ≤ R ≤ J
A[R] = X for J < R < I
A[R] ≥ X for I ≤ R ≤ N

```

The procedure uses an integer procedure random (M,N) which chooses equiprobably a random integer F between M and N, and also a procedure exchange, which exchanges the values of its two parameters;

```

begin real X; integer F;
      F := random (M,N); X := A[F];
      I := M; J := N;
up:    for I := I step 1 until N do
      if X < A [I] then go to down;
      I := N;
down:  for J := J step -1 until M do
      if A[J] < X then go to change;
      J := M;
change: if I < J then begin exchange (A[I], A[J]);
      I := I + 1; J := J - 1;
      go to up
      end
else if I < F then begin exchange (A[I], A[F]);
      I := I + 1
      end
else if F < J then begin exchange (A[F], A[J]);
      J := J - 1
      end;
end partition

```

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PARTITION, QUICKSORT, FIND [C. A. R. Hoare,  
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The body of the procedure find was corrected to read:

```

begin integer I, J;
if M < N then begin partition (A, M, N, I, J);
      if K ≤ I then find (A, M, J, K)
      else if J ≤ K then find (A, I, N, K)
      end
end find

```

and the trio of procedures was then successfully run using the Elliott ALGOL translator on the National-Elliott 803.

The author's estimate of  $\frac{1}{3}(N-M) \ln(N-M)$  for the number of

exchanges required to sort a random set was found to be correct. However, the number of comparisons was generally less than  $2(N-M) \ln(N-M)$  even without the modification mentioned below.

The efficiency of the procedure quicksort was increased by changing its body to read:

```

begin integer I, J;
if M < N-1 then begin partition (A, M, N, I, J);
      quicksort (A, M, J);
      quicksort (A, I, N)
      end
else if N-M = 1 then begin if A[N] < A[M] then
      exchange (A[M], A[N])
      end
end quicksort

```

**end** quicksort

This alteration reduced the number of comparisons involved in sorting a set of random numbers by 4-5 percent, and the number of entries to the procedure partition by 25-30 percent.

CERTIFICATION OF ALGORITHMS 63, 64 AND 65,  
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Algorithms 63, 64, and 65 have been tested using the Pegasus ALGOL 60 Compiler developed at the De Havilland Aircraft Company Ltd., Hatfield, England.

No changes were necessary to Algorithms 63 and 64 (Partition and Quicksort) which worked satisfactorily. However, the comment that Quicksort will sort an array without the need for any extra storage space is incorrect, as space is needed for the organization of the sequence of recursive procedure activations, or, if implemented without using recursive procedures, for storing information which records the progress of the partitioning and sorting.

A misprint ('if' for 'if' on the line starting 'else if J ≤ K then ...') was corrected in Algorithm 65 (Find), but it was found that in certain cases the sequence of recursive activations of Find would not terminate successfully. Since Partition produces as output two integers J and I such that elements of the array A[M:N] which lie between A[J] and A[I] are in the positions that they will occupy when the sorting of the array is completed, Find should cease to make further recursive activations of itself if K fulfills the condition J < K < I.

Therefore the conditional statement in the body of Find was changed to read

```

if K ≤ J then find (A, M, J, K)
else if I ≤ K then find (A, I, N, K)

```

With this change the procedure worked satisfactorily.