ALGORITHM 89

EVALUATION OF THE FRESNEL SINE INTEGRAL John L. Cundiff

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real procedure FRESNELSIN (u) Result: (frsin); value u; comment This algorithm computes the Fresnel sine integral defined by,

$$S(u) = \int_0^u \sin \pi t^2/2 \ dt,$$

by evaluating the series expansion

$$S(x) = \sqrt{\frac{2x}{\pi}} \left[\frac{x}{3} - \frac{x^3}{7 \cdot 3!} + \frac{x^5}{11 \cdot 5!} - \frac{x^7}{15 \cdot 7!} + \cdots \right]$$

where $x = \pi u^2/2$. Reference: Pearcey, T. Table of the Fresnel Integral to Six Decimal Places. The Syndics of the Cambridge University Press, Melbourne, Australia (1956).;

begin Pi2 := 1.5707963; $x := Pi2 \ x \ (u \uparrow 2)$; frsin := x/3; frsqr := $x \uparrow 2$; N := 3; term := $(-x \times frsqr)/6$; frsini := frsin + term/7;

if frsin = frsini then go to exit; frsin := frsini; Loop: $term := -term \times frsqr/((2{\times}N{-}1) \times (2{\times}N{-}2));$ frsini := frsin + term/ $(4 \times N - 1)$; N := N + 1; go to Loop;

frsin := frsini × u exit: end FRESNELSIN;

REMARK ON ALGORITHMS 88, 89 AND 90 EVALUATION OF THE FRESNEL INTEGRALS [J. L. Cundiff, Comm. ACM, May 1962]

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While coding these algorithms in Fortran for the IBM 7094, modifications were required (both in the formulation and in the language) before execution with any degree of speed and accuracy could be obtained. In the process it was found that the reference, Pearcy, contains an error in the formula for C(u). This error is contained in Algorithm 88 in the formula

$$C(u) = \frac{1}{2} - \frac{\sin(x)}{\sqrt{2\pi x}} [] - \cdots.$$

The first minus sign above should be a plus sign.

After the necessary modifications were made, the three algorithms were found to be too large and uneconomical for our usage. A single algorithm, incorporating these three procedures, was written and is in current usage in a computer program which requires several thousand evaluations of each Fresnel integral.