

# PRACNIQUES

The Techniques Department is interested in publishing short descriptions of Techniques which improve the logistics of information processing. To quote from the policy statement, Communications of the ACM 1 (Jan. 1958), 5: "It is preferable that techniques contributed be factual and in successful usage, rather than speculative or theoretical. One of the major criteria for acceptance and the question one should answer before submitting any material is—Can the reader use this tomorrow?" Clear, concise statements of fairly well-known but rarely documented methods will contribute significantly to raising the general level of professional competence.—C.L.McC.

## ON THE INVERSE OF A TEST MATRIX

In reference to the test matrix for determinants and inverses suggested by John Caffrey [*Comm. ACM* 6, 6 (June 1963)], it is possible to state the elements of the inverse explicitly. The test matrix was given as

$$A = a_{ij} = k \begin{pmatrix} i + j - 2 \\ i - 1 \end{pmatrix}$$

where  $k$  is an arbitrary nonzero constant.

The inverse matrix is

$$V = v_{ij} = \frac{(-1)^{i+j}}{k} \sum_{g=\max(i,j)}^n \binom{g-1}{i-1} \binom{g-1}{j-1}$$

where  $n$  is the order of the matrix. The inverse elements can be computed recursively as follows:

$$v_{in} = v_{ni} = \frac{(-1)^{n+i}}{k} \binom{n-1}{i-1};$$

$$v_{ij} = v_{i+1,j} + v_{i,j+1} + \frac{(-1)^{i+j}}{k} \binom{n}{i} \binom{n}{j},$$

for  $1 \leq i, j \leq n-1$ .

Let  $V^{(n)}$  denote the inverse of order  $n$ . Then for fixed  $k$ ,  $V^{(n+1)}$  can be generated from  $V^{(n)}$ :

$$v_{11}^{(n+1)} = \frac{n+1}{k};$$

$$v_{1,n+1}^{(n+1)} = v_{n+1,1}^{(n+1)} = \frac{(-1)^n}{k};$$

$$v_{n+1,n+1}^{(n+1)} = \frac{1}{k};$$

$$v_{i1}^{(n+1)} = v_{1i}^{(n+1)} = v_{i1}^{(n)} - v_{i-1,1}^{(n)} \quad \text{for } 2 \leq i \leq n;$$

$$v_{i,n+1}^{(n+1)} = v_{n+1,i}^{(n+1)} = v_{i-1,n}^{(n)} - v_{i,n}^{(n)}, \quad \text{for } 2 \leq i \leq n;$$

$$v_{ij}^{(n+1)} = v_{ij}^{(n)} + v_{i-1,j-1}^{(n)} - v_{i,j-1}^{(n)} - v_{i-1,j}^{(n)},$$

for  $2 \leq i, j \leq n$ .

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J. H. WEGSTEIN, Editor

## ALGORITHM 207

### STRINGSORT

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**procedure** stringsort ( $a, n$ ); **comment** elements  $a[1] \cdots a[n]$  of  $a[1:2n]$  are sorted into ascending sequence using  $a[n+1] \cdots a[2n]$  as auxiliary storage. Von Neumann extended string logic is employed to merge input strings from both ends of a sending area into output strings which are sent alternately to either end of a receiving area. The procedure takes advantage of naturally occurring ascending or descending order in the original data;

**value**  $n$ ; **integer**  $n$ ; **array**  $a$ ;

**begin integer**  $d, i, j, m, u, v, z$ ; **integer array**  $c[-1:1]$ ;

**switch**  $p := jz1, str i$ ; **switch**  $q := merge, jz2$ ;

oddpass:  $i := 1$ ;  $j := n$ ;  $c[-1] := n + 1$ ;  $c[1] := 2 \times n$ ;

allpass:  $d := 1$ ; **go to** firststring;

merge: **if**  $a[i] \geq a[z]$

**then begin go to**  $p[v]$ ;

$jz1$ : **if**  $a[j] \geq a[z]$

**then**  $ij$ : **begin if**  $a[i] \geq a[j]$

**then**  $str j$ : **begin**  $a[m] := a[j]$ ;

$j := j - 1$  **end**

**else**  $str i$ : **begin**  $a[m] := a[i]$ ;

$i := i + 1$  **end**

**end**

**else begin**  $v := 2$ ; **go to**  $str i$  **end**

**end**

**else begin**  $u := 2$ ;

$jz2$ : **if**  $a[j] \geq a[z]$

**then go to**  $str j$

**else begin**  $d := -d$ ;  $c[d] := m$ ;

firststring:  $m := c[-d]$ ;

$v := u := 1$ ;

**go to**  $ij$

**end**

**end**;

$z := m$ ;  $m := m + d$ ; **if**  $j \geq i$  **then go to**  $q[u]$ ;

**if**  $m > n + 1$  **then begin comment** evenpass;  $i := n + 1$ ;

$j := 2 \times n$ ;  $c[-1] := 1$ ;  $c[1] := n$ ; **go to** allpass **end**

**allpass end**

**else if**  $m < n + 1$  **then go to** oddpass

**end stringsort**;

## ALGORITHM 208

### DISCRETE CONVOLUTION

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**procedure** Discrete Convolution ( $m, n, prs$ ) result: ( $Conv$ );

**integer**  $m, n$ ; **real procedure**  $prs$ ; **real array**  $conv$ ;

**comment** This procedure finds the probability distribution of the sum of  $m$  independent variables, each with a known distribu-

tion over the nonnegative integers. A real procedure *prs* with results *pr*[*k*] is assumed to find each probability distribution in succession. The maximum sum for which probabilities are computed must be fixed by the user. The number of iterations is roughly  $m^2n/2$ . The procedure *prs* will in general depend on additional parameters and should include the read-in of the parameters for that distribution. It may include the selection of one function from a set;

```

begin integer i, j, k, ix1, ix2;
real array prob [1:2, 0:m], pr[0:m];
i := 1; ix1 := 1; ix2 := 2; prs (m) result: (pr);
for j := 0 step 1 until m do prob[ix1, j] := pr[j];
for i := 2 step 1 until n do
  begin
    if ix1 = 1 then begin ix2 := 1; ix1 := 2 end
    else begin ix2 := 2; ix1 := 1 end
    prs (m) result: (pr);
    for j := 0 step 1 until m do
      begin
        prob[ix1, j] := 0;
        for k := 0 step 1 until j do
          prob[ix1, j] := prob[ix1, j] + pr[k] × prob[ix2, j-k]
        end j
      end i;
  for j := 0 step 1 until m do conv[j] := prob[ix1, j]
end Discrete Convolution
comment The convolution of discrete probability series is isomorphic to the multiplication of polynomials. A useful variation is to omit the parameters i, n and have prs recognize the end of input. A FORTRAN program using this procedure has been run on the IBM 7090 to find the sum of queue lengths in a teletype switching center, where messages arrived according to the Poisson distribution and message lengths were distributed negative-exponentially. The following was used as the probability procedure;
procedure prs (m) result: (pr);
value m; procedure read;
real array pr; integer m;
begin real trafficrate, linespeed, rho; integer j;
  read (trafficrate, linespeed);
  rho := trafficrate/linespeed;
  pr[0] := 1 - rho;
  for j := 1 step 1 until m do pr[j] := rho × pr[j-1]
end prs

```

A contribution to this department must be in the form of an Algorithm, a Certification, or a Remark. Contributions should be sent in duplicate to the Editor and should be written in a style patterned after recent contributions appearing in this department. An algorithm must be written in ALGOL 60 (see *Communications of the ACM*, January 1963) and accompanied by a statement to the Editor indicating that it has been tested and indicating which computer and programming language was used. For the convenience of the printer, contributors are requested to double space material and underline delimiters and logical values that are to appear in boldface type. Whenever feasible, Certifications should include numerical values.

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## ALGORITHM 209

### GAUSS

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```

real procedure Gauss(x); value x; real x;
comment Gauss calculates  $(1/\sqrt{2\pi})\int_{-\infty}^x \exp(-\frac{1}{2}u^2) du$  by means of polynomial approximations due to A. M. Murray of Aberdeen University;
begin real y, z, w;
  if x = 0 then z := 0
  else
    begin y := abs(x)/2;
      if y ≥ 3 then z := 1
      else if y < 1 then
        begin w := y × y;
          z := (((((((((0.000124818987 × w
            - 0.001075204047) × w + 0.005198775019) × w
            - 0.019198292004) × w + 0.059054035642) × w
            - 0.151968751364) × w + 0.319152932694) × w
            - 0.531923007300) × w + 0.797884560593) × y × 2
          end
        else
          begin y := y - 2;
            z := ((((((((((((-0.000045255659 × y
              + 0.000152529290) × y - 0.000019538132) × y
              - 0.000676904986) × y + 0.001390604284) × y
              - 0.000794620820) × y - 0.002034254874) × y
              + 0.006549791214) × y - 0.010557625006) × y
              + 0.011630447319) × y - 0.009279453341) × y
              + 0.005353579108) × y - 0.002141268741) × y
              + 0.000535310849) × y + 0.999936657524
            end
          end;
        Gauss := if x > 0 then (z+1)/2 else (1-z)/2
      end Gauss;

```

## ALGORITHM 210

### LAGRANGIAN INTERPOLATION

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```

procedure LAGRANGE (N, u, X, Y, ANS); real array X, Y;
integer N; real u, ANS;
comment This procedure evaluates an Nth degree Lagrange polynomial, given N + 1 data coordinates, and u the value where interpolation is desired. X is the abscissa array and Y the ordinate array. ANS is the resultant value of the function at u. The notation is that used in R. W. Hamming, Numerical Methods for Scientists and Engineers, pp. 94-95 (McGraw-Hill Book Company, Inc., 1962);
begin integer i, j; real L;
  ANS := 0.0;
  for j := step 1 until N+1 do
    begin L := 1.0;
      for i := step 1 until N+1 do
        begin if i ≠ j then L := L × (u-X[i])/(X[j]-X[i])
        end;
      ANS := ANS + L × Y[j]
    end j
end end

```

ALGORITHM 211  
HERMITE INTERPOLATION

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**procedure** *HERMITE* (*n, u, X, Y, Y1, ANS*); **real array** *X, Y, Y1*;

**integer** *n*; **real** *u, ANS*;

**comment** This procedure evaluates a  $(2n+1)$ th degree Hermite polynomial, given the value of the function and its first derivative at each of  $n+1$  points. *X* is the abscissa array, *Y* the ordinate array, and *Y1* the derivative array. *ANS* is the interpolated value of the function at *u*. **REFERENCE:** R. W. Hamming, *Numerical Methods for Scientists and Engineers*, pp. 96-97 (McGraw-Hill Book Company, Inc., 1962);

**begin integer** *i, j*; **real** *h, a*;

*ANS* := 0.0;

**for** *j* := 1 **step** 1 **until**  $n+1$  **do**

**begin** *h* := 1.0; *a* := 0.0;

**for** *i* := 1 **step** 1 **until**  $n+1$  **do**

**begin if** *i* = *j* **then go to** *out*;

*h* := *h* × (*u* - *X*[*i*])↑2 / (*X*[*j*] - *X*[*i*])↑2;

*a* := *a* + 1.0 / (*X*[*j*] - *X*[*i*]);

*out*: **end**;

*ANS* := *ANS* + *h* × ((*X*[*j*] - *u*) × (2 × *a* × *Y*[*j*] - *Y1*[*j*] + *Y*[*j*]))

**end end**

ALGORITHM 212  
FREQUENCY DISTRIBUTION

MALCOLM D. GRAY

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**procedure** *FREQUENCY* (*N, A, B, IUL, K, X, KA*);

**integer** *N, IUL*; **integer array** *KA*; **real** *A, B, K*;

**real array** *X*;

**comment** Given a set *X* of variables in some interval  $I = [a, b]$  such that  $a \leq \min x, \max x \leq b$ , *FREQUENCY* determines the frequency distribution of *X* over  $k$  equal, half open subintervals of *I*. The interval *I* is transformed to the interval  $J = [0, k]$  with unit subintervals by  $x' = (x_i - a) / [(b - a) / k]$ ,  $i = 1, 2, \dots, n$ , and considering  $x' = L \times M$ , *L* and *M* integers. The value *L* then immediately determines the subinterval and *M* is used for boundary points. If *IUL* = 0, the subintervals are open on the upper end, except the *k*th. On entry, the array *KA* is assumed identically zero; on return, *KA*[*i*] contains the frequency of *X* in the *i*th subinterval;

**begin integer** *i, L*; **real** *BAK, XP*;

*BAK* := (*B* - *A*) / *K*;

**for** *i* := 1 **step** 1 **until** *N* **do begin**

*XP* := (*X*[*i*] - *A*) / *BAK*;

*L* := *entier* (*XP*);

**if** *XP* = *L* **then go to** *p2* **else** *L* := *L* + 1; **go to** *p5*;

*p2*: **if** *IUL* = 0 **then go to** *p3* **else if** *L* = 0 **then** *L* := *L* + 1; **go to** *p5*;

*p3*: **if** *XP* ≠ *K* **then** *L* := *L* + 1;

*p5*: *KA*[*L*] := *KA*[*L*] + 1;

**end**;

**end** *FREQUENCY*

ALGORITHM 213  
FRESNEL INTEGRALS

MALCOLM D. GRAY

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**real procedure** *FRESNEL* (*w, S, C*); **value** *w*; **real** *S, C*;

**comment** *FRESNEL* computes the Fresnel sine and cosine in-

tegrals  $S(w) = \int_0^w \sin [(\pi/2)t^2] dt$  and  $C(w) = \int_0^w \cos [(\pi/2)t^2] dt$  using the series expansions

$$S(w) = w \sum_{i=1}^{\infty} \frac{(-1)^{i+1} x^{2i-1}}{(4i-1)(2i-1)!} \quad \text{and}$$

$$C(w) = w \sum_{i=1}^{\infty} \frac{(-1)^{i+1} x^{2i-2}}{(4i-3)(2i-2)!}$$

for  $|w| < \sqrt{22/\pi}$  and  $x \equiv \pi w^2/2$ , and using the asymptotic series

$$S(w) = \alpha - \frac{1}{\pi w} [P(x) \sin(x) + Q(x) \cos(x)],$$

$$C(w) = \alpha - \frac{1}{\pi w} [P(x) \cos(x) - Q(x) \sin(x)]$$

where  $|w| \geq \sqrt{22/\pi}$ ,  $x \equiv \pi w^2/2$ ,

$$Q(x) = 1 - \sum_{i=2}^{\infty} \frac{(-1)^i (4i-5)!!}{(2x)^{2i-2}}, \quad P(x) = \sum_{i=1}^{\infty} \frac{(-i)^{i+1} (4i-3)!!}{(2x)^{2i-1}},$$

and  $n!! \equiv n(n-2)(n-4)\dots 1$ . If  $w \geq 0$ , then  $\alpha = \frac{1}{2}$ , or if  $w < 0$ , then  $\alpha = -\frac{1}{2}$ .

This algorithm is a translation of a FAP coded subroutine currently in use on the IBM 7094 at the Boeing Company. The FAP program yields the following errors when tested at 0.05 increments of  $x$ :

<i>x</i>	$\Delta S$	$\Delta C$
0.00, 1.00	$< 1 \times 10^{-7}$	$< 1 \times 10^{-7}$
1.05, 8.65	$< 1 \times 10^{-6}$	$< 1 \times 10^{-6}$
8.70, 10.30	$3 \times 10^{-6}$	$2 \times 10^{-6}$
10.35, 11.00	$5 \times 10^{-6}$	$4 \times 10^{-6}$
11.05, 12.15	$< 1 \times 10^{-6}$	$3 \times 10^{-6}$
12.20, 15.00	$< 1 \times 10^{-6}$	$< 1 \times 10^{-6}$

where  $\Delta S$  and  $\Delta C$  are the approximate average absolute deviations (over the range) from the reference. The user must supply  $S(w) = C(w) = \pm \frac{1}{2}$  if  $w \rightarrow \pm \infty$ . **REFERENCES:** ALGORITHMS 88-90, J. L. Cundiff, *Comm. ACM*, May 1962; Born, M. and Wolf, E., *Principles of Optics*, Pergamon Press (1958), pp. 369-431;

**begin real** *x, x2, eps, term*; **integer** *n*; *eps* := 0.000001;  
*x* := *w* × *w* / 0.6366198;  
*x2* := -*x* × *x*; **if** *x* ≥ 11.0 **then go to** *asympt*;  
**begin real** *frs, frsi*;  
*frs* := *x* / 3; *n* := 5; *term* := *x* × *x2* / 6;  
*frsi* := *frs* + *term* / 7;  
*loops*: **if**  $\text{abs}(\text{frs} - \text{frsi}) \leq \text{eps}$  **then go to** *send*; *frs* := *frsi*;  
*term* := *term* × *x2* / *n* / (*n* - 1); *frsi* := *frs* + *term* / (*n* + *n* + 1);  
*n* := *n* + 2; **go to** *loops*;  
*send*: *S* := *frsi* × *w*; **end**;  
**begin real** *frc, frci*;  
*frc* := 1; *n* := 4; *term* := *x2* / 2; *frci* := 1 + *term* / 5;  
*loopc*: **if**  $\text{abs}(\text{frc} - \text{frci}) \leq \text{eps}$  **then go to** *send*; *frc* := *frci*;  
*term* := *term* × *x2* / *n* / (*n* - 1); *frci* := *frc* + *term* / (*n* + *n* + 1);  
*n* := *n* + 2; **go to** *loopc*;  
*send*: *C* := *frci* × *w*; **end**; **go to** *aend*;  
*asympt*: **begin real** *S1, S2, half, temp*; **integer** *i*;  
*x2* := 4 × *x2*; *term* := 3 / *x2*; *S1* := 1 + *term*; *n* := 8;  
**for** *i* := 1 **step** 1 **until** 5 **do begin** *n* := *n* + 4;  
*term* := *term* × (*n* - 7) × (*n* - 5) / *x2*; *S1* := *S1* + *term*;  
**if**  $\text{abs}(\text{term}) \leq \text{eps}$  **then go to** *next*; **end** *i*;  
*next*: **for** *i* := 1 **step** 1 **until** 5 **do begin** *n* := *n* + 4;  
*term* := *term* × (*n* - 5) × (*n* - 3) / *x2*; *S2* := *S2* + *term*;  
**if**  $\text{abs}(\text{term}) \leq \text{eps}$  **then go to** *final*; **end** *i*;  
*final*: **if** *w* < 0 **then** *half* := -0.5 **else** *half* := 0.5; *term* :=  $\cos(x)$ ;  
*temp* :=  $\sin(x)$ ; *x2* := 3.1415927 × *w*;  
*C* := *half* + (*temp* × *S1* - *term* × *S2*) / *x2*;  
*S* := *half* - (*term* × *S1* + *temp* × *S2*) / *x2*;  
**end**;  
*aend*: **end** *FRESNEL*

CERTIFICATION OF ALGORITHM 27  
 ASSIGNMENT [Roland Silver, *Comm. ACM*, Nov. 1960]  
 ALBERT NEWHOUSE  
 University of Houston, Houston, Texas

The ASSIGNMENT algorithm was translated into MAD and successfully run on the IBM 709/7094 after the following corrections were made:

All references to array  $a$  and  $d$  refer to the same array, i.e. change all  $a[i, j]$  to  $d[i, j]$ . Furthermore:

(a) 3rd line after LABEL: **comment**: Label and scan; should read

**begin if**  $d[i, j] \neq 0 \vee \text{lambda}[j] \neq 0$  **then go**

(b) first line after J3: **end j**; should read

$\text{min} := d[r[1], cb[1]]$ ;

(c) line I2:

should read

I2: **for**  $l := 1$  **step 1 until**  $cbl$  **do**

Since there is no provision made for this algorithm to end the following additions were made:

(1) in the integer declaration add the variable:  $flag$

(2) first line after START: **comment**: ... add the line

$flag := n$ ;

(3) first line before I1: **end i**;

change to read

$rl := rl + 1$ ;  $r[rl] := i$ ;  $mu[i] := -1$ ;  $flag := flag - 1$

(4) add a line after I1: **end i**;

**if**  $flag = n$  **then go to FINI**;

(5) change the last line of the algorithm to read:

FINI: **end** Assignment

In order to obtain the minimum value of the  $\sum_{i=1}^n a_{ix_i}$  (in the following called total) the following additions may be made:

Add a real variable  $total$  and

(A) new line after INITIALIZE;

$total := 0$ ;

(B) new line after the first **end i**;

$total := total + \text{min}$ ;

(C) new line after the first **end j**;

$total := total + \text{min}$ ;

(D) after the line **end k**; after J3: **end j**;

add the line

$total := total + (rl + cbl - n) \times \text{min}$ ;

REMARK ON ALGORITHMS 88, 89 AND 90  
 EVALUATION OF THE FRESNEL INTEGRALS  
 [J. L. Cundiff, *Comm. ACM*, May 1962]

MALCOLM D. GRAY

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While coding these algorithms in FORTRAN for the IBM 7094, modifications were required (both in the formulation and in the language) before execution with any degree of speed and accuracy could be obtained. In the process it was found that the reference, *Pearcy*, contains an error in the formula for  $C(u)$ . This error is contained in Algorithm 88 in the formula

$$C(u) = \frac{1}{2} - \frac{\sin(x)}{\sqrt{2\pi x}} [ ] - \dots$$

The first minus sign above should be a plus sign.

After the necessary modifications were made, the three algorithms were found to be too large and uneconomical for our usage. A single algorithm, incorporating these three procedures, was written and is in current usage in a computer program which requires several thousand evaluations of each Fresnel integral.

REMARK ON ALGORITHM 123  
 ERF(x) [Martin Crawford and Robert Techo, *Comm. ACM*, Sept. 1962]

D. IBBETSON

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(1) The specification **value**  $x$ ; was added to allow  $x$  to be an expression and to prevent side effects.

(2) The algorithm was then modified to give the Gaussian integral  $(1/\sqrt{2\pi}) \int_{-\infty}^x \exp(-\frac{1}{2}u^2) du$  by

(a) changing its name to  $Gauss(x)$ ,

(b) inserting  $x := x * 0.70710678$ ; immediately before  $Z := 0$ ; , and

(c) changing the final statement to

$Gauss := (Z+1)/2$  **end Gauss**

(3) The algorithm with the above changes was tested on a National Elliott 803 computer using the Elliott-ALGOL translator with  $10^{-8}$  substituted for  $10^{-10}$ . It was found to produce wrong answers when  $x = \pm 1$  (corresponding to  $Erf(\pm 1/\sqrt{2})$ ) giving  $0.5 \pm 0.3467899$  instead of  $0.5 \pm 0.3413447$ .

REMARK ON ALGORITHM 157  
 FOURIER SERIES APPROXIMATION [Charles J. Mifsud, *Comm. ACM*, Mar. 1963]

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Algorithm 157 has been modified to fit  $2N$  data points and has run successfully on the Burroughs 220 using BALGOL. With the modifications,  $2N$  constants  $a_p$  ( $p=0, 1, \dots, N$ ) and  $b_p$  ( $p=1, 2, \dots, N-1$ ) are determined such that the equation  $f_n = a_0/2 + \sum_{p=1}^{N-1} (a_p \cos \pi np/N + b_p \sin \pi np/N) + a_N/2 \cos \pi n$  is satisfied.

In the modified procedure, the second and third lines after the integer declaration should read:

$C[1] := \cos(\pi i/N)$ ;

$S[1] := \sin(\pi i/N)$ ;

The second **for** statement should read:

**for**  $i := 2 \times N - 1$  **step**  $-1$  **until**  $1$  **do**

The lines containing the  $a$  and  $b$  coefficients should read:

$a[p] := (f[0] + u[1] \times C[2] - u[2])/N$ ;

$b[p] := (u[1] \times S[2])/N$ ;

REFERENCE: R. W. Hamming, *Numerical Methods for Scientists and Engineers*, pp. 68-73 (McGraw-Hill, 1962).

CERTIFICATION OF ALGORITHM 160  
 COMBINATORIAL OF M THINGS TAKEN N AT  
 A TIME [M. L. Wolfson and H. V. Wright, *Comm. ACM*, April 1963]

ROBERT F. BLAKELY

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Algorithm 160 was translated into ALGO, a compiler for the Control Data Corp. G-15 computer (formerly the Bendix G-15).

With the restriction that  $m \geq n \geq 0$ , correct results were obtained for all integer values of  $m$  and  $n$ , where  $0 \leq m \leq 10$ . Several other values were tested and all results were correct.

CERTIFICATION OF ALGORITHM 161  
 COMBINATORIAL OF M THINGS TAKEN ONE AT  
 A TIME, TWO AT A TIME, UP TO N AT A TIME  
 [H. V. Wright and M. L. Wolfson, *Comm. ACM*,  
 Apr. 1963]

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Algorithm 161 was translated into ALGO, a compiler for the Control Data Corp. G-15 computer (formerly the Bendix G-15).

With the restriction that  $m \geq n \geq 1$ , correct results were obtained for all integer values of  $m$  and  $n$ , where  $1 \leq m = n \leq 15$ . Several other values were tested (including cases where  $m \neq n$ ) and all results were correct.

CERTIFICATION OF ALGORITHM 173  
 ASSIGN [Otomar Hájek, *Comm. ACM*, June 1963]  
 R. S. SCOWEN

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Algorithm 173 (ASSIGN) has been tested successfully using the DEUCE ALGOL 60 compiler. The only changes necessary were the addition of specifications for the formal parameters  $a$ ,  $b$  (DEUCE ALGOL 60 compiler requires specifications for all formal parameters).

The author's example, *assign* ( $a[i[1], i[2]]$ , (if  $i[3]=1$  then 0.0 else  $a[i[1], i[2]]$ ) +  $b[i[1], i[3]] \times c[i[3], i[2]]$ , 3,  $i[j]$ , 1, if  $j = 1$  then  $n$  else if  $j = 2$  then  $m$  else  $p, j$ ); did form the matrix product  $B \times C$  and store it in  $A$ .

The algorithm was also used to read a matrix into the computer using the procedure call

```
assign (b[i[1], i[2]], read, 2, i[j], 1,
if j = 1 then n else p, j);
```

(*read* is a real procedure which takes the value given by the next number on the input tape).

These examples took about three times as long to run as the simpler equivalent statements

```
for i := 1 step 1 until n do
for j := 1 step 1 until m do
begin
a[i, j] := 0.0;
for k := 1 step 1 until p do
a[i, j] := a[i, j] + b[i, k] × c[k, j]
end;
```

and

```
for j := 1 step 1 until p do
for i := 1 step 1 until n do
b[i, j] := read;
```

CERTIFICATION OF ALGORITHM 173  
 ASSIGN [O. Hájek, *Comm. ACM*, July 1963]  
 Z. FILSAK and L. VRCHOVECKÁ

Research Institute of Mathematical Machines, Prague,  
 and Computing Center Kancelářské stroje, Prague

The algorithm was modified for input to the Elliott-ALGOL system as follows. In Elliott-ALGOL, name-called parameters in recursive procedures are prescribed. Luckily, the only parameter which varies during the recursive call in the body of *Assign* is called by value (it is the parameter *dim* which determines depth of recursion). The body of *Assign* was replaced by (i) a procedure declaration *Ass(dim)*, whose body is that of the original *Assign*, but with the recursive call of *Assign* replaced by that of *Ass*, and (ii) a single statement, the activation of *Ass(dim)*.

The resulting procedure was tested (on the National-Elliott 803 in the Computing Center), on a rather large set of examples, including those described in the text following Algorithm 173. It was found that in the last example, matrix multiplication, indices  $i_1$  and  $i_3$  should be interchanged throughout.

No changes of the algorithm itself were necessary. It seems that the modification described above, motivated by limitations of Elliott-ALGOL, also improve efficiency, at least for large dimensions of the arrays concerned.

CERTIFICATION OF ALGORITHM 175  
 SHUTTLE SORT [C. J. Shaw and T. N. Trimble, *Comm. ACM*, June 1963]

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\* Undergraduate research project, Computer Science Program, Univ. of Dayton.

Algorithm 175 was translated into BALGOL and ran successfully on the Burroughs 220. The following actual sorting times were observed:

Number of Items	Average Time (sec)
25	1.6
50	6.2
100	25.8
250	181
500	684

The algorithm can be extended so that the sort is made on one array, while retaining a one-to-one correspondence to a second array. This is done by inserting immediately before **end** of the  $j$  loop the following:

*Temporary* :=  $Y[j]$ ;  $Y[j]$  :=  $Y[j + 1]$ ;  $Y[j + 1]$  := *Temporary*; where  $Y[k]$  is the element to be associated with  $N[k]$ . Other variations are obviously possible.

CERTIFICATION OF ALGORITHM 210  
 HERMITE INTERPOLATION [George R. Schubert,  
*Comm. ACM*, Oct. 1963]

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The body of *HERMITE* was transcribed for the Dartmouth SCALP processor for the LGP-30 computer and ran successfully without corrections. It was tested using the error function and its derivatives. Roundoff error in the LGP-30 began to appear for values of  $n$  greater than 3. For  $n$  equal to 2 (third degree polynomial) the interpolated value agreed with the function within machine limitations (six significant figures) for steps in the argument data of 0.005.

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DATES TO REMEMBER

FJCC	Las Vegas	Nov. 12-14, 1963
SJCC	Washington	Apr. 21-23, 1964
ACM	Philadelphia	Aug. 25-28, 1964
FJCC	San Francisco	Nov. 17-19, 1964
IFIP	New York	May 22-24, 1965
ACM	Cleveland	Aug. 23-26, 1965

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