ALGORITHM 313
MULTI-DIMENSIONAL PARTITION GENERATOR [A1]

P. Bratley and J. K. S. McKay (Recd. 23 Aug. 1966, 15 Feb. 1967 and 14 Apr. 1967)
Dept. of Computer Science, University of Edinburgh

```
procedure partition (N, dim, use);
  value N, dim;  integer N, dim;  procedure use;
comment A partition of N is an ordered sequence of positive
```

integers, $n_1 \geq n_2 \geq n_3 \geq \cdots \geq n_k$, such that $\sum_{i=1}^{k} n_i = N$. Such a partition may be represented by a Ferrers-Sylvester graph of nodes with n_i nodes in the ith row, e.g.,

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* * * * *
* * * *
* *
* *
```

represents 5, 4, 2, 2. This two-dimensional diagram may be generalized in a natural way to three, or more, dimensions. More formally, we regard a d-dimensional partition of n as a set S of n nodes, each defined by its non-negative integer coordinates such that

$(x_1, x_2, \cdots, x_d) \in S$ if and only if $(x_1', x_2', \cdots, x_d') \in S$

whenever

$$0 \leq x_i' \leq x_i \quad \text{for all} \quad i = 1, 2, \cdots, d.$$

This generalization reduces to the usual definition when $d = 2$. There is little literature on these generalized partitions. It is with a view to facilitating numerical studies that this algorithm is published.

After generation, each partition is presented to the procedure *use*, which should be supplied by the user for the purpose he requires. *use* has three formal parameters, the first being the name of a two-dimensional integer array, and the second and third being integers giving the size of this array. When the procedure is called by

use (*current, dim, N*)

then the coordinates of the nodes entering into the newly generated multi-dimensional partition will be found in *current* [1:*dim*,1:*N*]. The parameters of *use* should be called by value, or alternatively care should be taken that neither *dim, N,* nor the contents of the array *current* are disturbed.

References:
1. Gupta, H., Gwyther, C. E., and Miller, J. C. P. *Tables of Partitions.* Royal Society Mathematical Tables, Vol. 4, Cambridge Univ. Press, 1958.
2. MacMahon, P. A. *Combinatory Analysis*, Vol. 2, Cambridge Univ. Press, 1916.
3. Chaundy, T. W. Partition generating functions. *Quart. J. Math. 2* (1931), 234–240.
4. Atkin, A. O. L., Bratley, P., MacDonald, I. G., and McKay, J. K. S. Some computations for m-dimensional partitions. *Proc. Cambridge Phil. Soc.* (to appear);

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begin
  integer i;  integer array current [1:dim, 1:N],
    x[1:dim,0:(N−1)×dim];
  procedure part (n,q,r);  value n, q, r;  integer n, q, r;
  begin integer s, i, j, k, p, m, z;
    for p:= q step 1 until r − 1 do
    begin
      for i := 1 step 1 until dim do current [i,n] := x[i,p];
      if n = N then begin use (current,dim,N);  go to L2 end;
      s := r;
      for i := 1 step 1 until dim do
      begin
        for j := 1 step 1 until dim do x[j,s] := x[j,p];
        x[i,s] := x[i,s] + 1;
        for j := 1 step 1 until dim do
        begin
          if x[j, s] = 0 then go to L3;
          for k := 1 step 1 until n do
          begin
            for m := 1 step 1 until dim do
            begin
              z := if j = m then 1 else 0;
              f ciurrent [m, k] ≠ x[m,s] − z then go to L4
            end;
            go to L3;
L4:
          end k;
          go to L5;
L3:
        end j;
        s := s + 1;
L5:
      end i;
      part (n+1,p+1,s);
L2: end p
  end part;
  for i := 1 step 1 until dim do x[i,0] := 0;   part (1,0,1)
end partition
```

REMARK ON CORRECTION TO CERTIFICATION OF ALGORITHM 279 [D1]
CHEBYSHEV QUADRATURE [F.R.A. Hopgood and C. Litherland, *Comm. ACM 9* (Apr. 1966), 270 and *10* (May 1967), 294]

Kenneth Hillstrom (Recd. 26 June 1967)
Applied Mathematics Division, Argonne National Laboratory, Argonne, Illinois

There are two corrections that should be appended to the certification of Algorithm 279.

Due to programming error, the integrand function routines for e^{-x^2} and $sin(x)+1$, used by the Chebyshev routine, incorrectly evaluated the functions at $x = 0$, thus delaying convergence.

The revised Chebyshev routine still converges more rapidly than the original scheme in the first two examples, but the advantage is much less pronounced than previously indicated.

The amended Table I should read as follows, with the numerical corrections italicized.

TABLE I

Integrand	*A*	*B*	*EPS*	*VI*	*Routine*	*VA*	*Number of function evaluations*
e^{-x^2}	0	4.3	10^{-6}	0.886226924	Havie	0.886226924	17
					Romberg	0.886226925	65
					Chebyshev	*0.8862269261*	*33*
					Chebyshev (Rev.)	0.8862269258	17
$sin(x)+1$	0	2π	10^{-6}	6.283185308	Havie	*6.283185307*	*3*
					Romberg	*6.283185307*	*3*
					Chebyshev	*6.2831853086*	*9*
					Chebyshev (Rev.)	6.2831853089	5