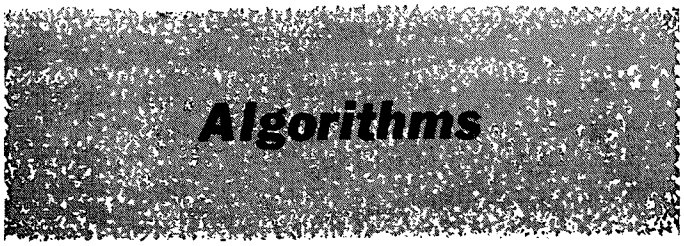

J. G. HERRIOT, Editor

ALGORITHM 334
NORMAL RANDOM DEVIATES [G5]

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KEY WORDS AND PHRASES: normal deviates, normal distribution, random number, random number generator, simulation, probability distribution, frequency distribution, random
CR CATEGORIES: 5.5, 5.13

```
procedure norm (D1, D2);
  real D1, D2;
comment This procedure generates pairs of independent
  normal random deviates with mean zero and standard deviation
  one. The output parameters D1 and D2 are normally distributed
  on the interval (−∞, +∞). The method is exact even in the
  tails.
  This algorithm is one of a class of normal deviate generators,
  which we shall call "chi-squared projections" [1, 2]. An al-
  gorithm of this class has two stages. The first stage selects a
  random number L from a χ2²-distribution. The second stage
  calculates the sine and cosine of a random angle θ. The generated
  normal deviates are given by L sin (θ) and L cos (θ).
  The two stages can be altered independently. In particular,
  as better χ2² random generators are developed, they can replace
  the first stage. (The negative exponential distribution is the
  same as that of χ2².)
  The fastest exact method previously published is Algorithm
  267 [4], which includes a comparison with earlier algorithms.
  It is a straight chi-squared projection. Our algorithm differs
  from it by using von Neumann rejection to generate sin (φ) and
  cos (φ), [φ = 2θ], without generating φ explicitly [3]. This
  significantly enhances speed by eliminating the calls to the
  sin and cos functions.
  The author wishes to express his gratitude to Professor
  George Forsythe for his help in developing the algorithm.
  REFERENCES
1. BOX, G., AND MULLER, M. A note on the generation of normal
   deviates. Ann. Math. Stat. 28, (1958), 610.
2. MULLER, M. E. A comparison of methods for generating
   normal deviates on digital computers. J. ACM, 6 (July
   1959), 376–383.
3. VON NEUMANN, J. Various techniques used in connection with
   random digits. In Nat. Bur. of Standards Appl. Math. Ser.
   12, 1959, p. 36.
4. PIKE, M. C. Algorithm 267, Random Normal Deviate.
   Comm. ACM, 8 (Oct. 1965), 606.;
comment R is any parameterless procedure returning a random
  number uniformly distributed on the interval from zero to one.
  A suitable procedure is given by Algorithm 266, Pseudo-Random
  Numbers [Comm. ACM, 8 (Oct. 1965), 605] if one chooses
  a = 0, b = 1, and initializes y to some large odd number, such
  as y = 13421773.;
begin
  real X, Y, XX, YY, S, L;
  comment von Neumann rejection for choosing a random
    angle φ = 2θ, θ = tan⁻¹ (Y/X);
A: X := R; Y := 2 × R − 1;
  XX := X ↑ 2; YY := Y ↑ 2;
  S := XX + YY;
  if S > 1 then go to A;
  comment chooses L randomly from a χ2²-distribution and
    normalizes with S;
L := sqrt (−2×ln(R))/S;
  comment computes deviates as L × sin (φ) and L × cos (φ);
D1 := (XX−YY) × L;
D2 := 2 × X × Y × L;
end norm;
```

REMARK ON ALGORITHM 178 [E4]
DIRECT SEARCH [Arthur F. Kaupe, Jr., *Comm. ACM 6* (June 1963), 313]

[as revised by M. Bell and M. C. Pike, *Comm. ACM 9* (Sept. 1966), 684]

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KEY WORDS AND PHRASES: function minimization, search, direct search
CR CATEGORIES: 5.19

The procedure does not exit, as specified, after *maxeval* (the maximum number of) function evaluations.

The 3 statements *eval* := *eval* +1 should be interchanged with the immediately preceding statement and replaced by a call to the procedure *test eval* defined below. The statement labeled 2 should be deleted.

```
procedure test eval;
  if eval < maxeval then eval := eval +1
  else begin converge := false;
  go to EXIT
  end test eval
```

REMARK ON ALGORITHM 272
PROCEDURE FOR THE NORMAL DISTRIBUTION FUNCTIONS [S15] [M. D. MacLaren, *Comm. ACM 8* (Dec. 1965), 789]

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KEY WORDS AND PHRASES: normal distribution function, error function, normal function, normal curve integral
CR CATEGORIES: 5.5, 5.12

In [1] Hill and Joyce report that the value produced by Algorithm 272 for the argument $a = 0.8$ is correct only to 5 decimal places, although the algorithm specifies an accuracy of 2×10^{-8}. Upon checking we have found that the source of this inaccuracy is a typographical error in the section beginning "**begin comment** initialize **own** variables;" The statement initializing $C[3]$ should be changed to "$C[3] = .54674530$." With this change the published algorithm is, as far as we know, accurate within the specified error limit of 2×10^{-8}.

In the first comment of the algorithm the lower limit of the first integral should be minus infinity and not merely a minus sign.

REFERENCE:

1. HILL, I. D., AND JOYCE, S. A. Remark on algorithm 123. *Comm. ACM 10* (June 1967), 377.